



Application of system identification techniques to aircraft gas turbine engines[☆]

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Abstract

A variety of system identification techniques are applied to the modelling of aircraft gas turbine dynamics. The motivation behind the study is to improve the efficiency and cost-effectiveness of system identification techniques currently used in the industry. Three system identification approaches are outlined in this paper. They are based upon: multisine testing and frequency-domain identification, time-varying models estimated using extended least squares with optimal smoothing, and multiobjective genetic programming to select model structure. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Gas turbines; System identification; Frequency domain; Multisine signals; Least-squares estimation; Time-varying systems; Genetic algorithms; Structure selection

1. Introduction

The modelling of gas turbines is a topic of great importance, given their widespread use in aero, marine and industrial applications. Three system identification techniques are employed in this paper, with specific application to an aero gas turbine engine. The techniques span the statistical and systems approaches described by Ljung (1996) in a recent survey of system identification. The motivation behind this work was to reduce test times while improving and quantifying model accuracy. The project involved three university teams, in collaboration with a leading gas turbine manufacturer.

Aircraft gas turbines are subjected to rigorous tests immediately after manufacture, in order to ensure reliable operation and give confidence in design predic-

tions. One such test is a dynamic verification test, conducted on a static test-bed such as that shown in Fig. 1. The dynamic models obtained can then be used for simulation and as a basis for control system design. Engine dynamics arise from complex, interacting phenomena: gas-flow behaviour in the compressor and turbine (affected by air inlet as well as engine conditions), shaft inertias and losses, fuel-flow transport delay, fuel dispersal and combustion, and the thermal behaviour of the engine and its surroundings.

Identification of aircraft gas turbine dynamics has historically relied on the use of “wobble” tests, in which the engine is excited by single sines of different frequencies. The gain and phase shift are then calculated at each frequency, generating Bode plots which are easy to interpret and familiar to control engineers. Low-amplitude sinewave tests can be made insensitive to the nonlinearities in the engine response and to noise. However, they have severe drawbacks, the primary one being the expense associated with very long test durations to allow the decay of initial transients at each frequency.

Moreover, the presence of nonlinearities necessitates taking a frequency response at each of several operating conditions spanning the range of engine speeds from

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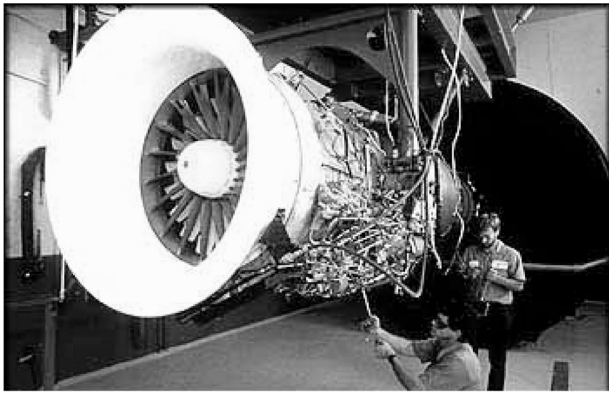


Fig. 1. Aero gas turbine test-bed.

ground idle to full power. There are at least two sources of nonlinearity in the engine response. The first of these is the nonlinear variation of the engine dynamics with operating power and condition (Saravanamuttoo, 1992). Secondly, the dynamics change with the thermal state of the engine; for instance they differ between a fast and slow acceleration, due to effects such as changes in blade-tip clearances (Pilidis & Maccallum, 1986). Some such effects can be treated as slow linear modes but the frequency responses give very limited insight into the slow thermal dynamics of the engine.

Finally, the current wobble-test procedure, although believed to give accurate estimation of the small-signal, steady-state engine behaviour, provides little indication of the accuracy of the results.

The work described here is intended to improve the speed and quality of engine testing and to allow better exploitation of test-bed results. The overall aim is to gain insight into alternatives to traditional testing and identification methods for multi-shaft aircraft gas turbine engines. This case study utilised a large set of engine records from a twin-shaft Rolls Royce Spey engine, logged by the Defence Evaluation and Research Agency (DERA) Pyestock, according to test schedules specified by the university teams.

This paper concentrates on the dynamic relationship between the measured input fuel flow and the high-pressure (HP) and low-pressure (LP) shaft speeds, denoted by N_H and N_L . The engine speed control was operated in open-loop and a perturbed fuel-demand signal was fed to the fuel-feed system, which regulates the fuel flow to the engine by means of a stepper valve, as shown in Fig. 2. The fuel flow was measured downstream of the fuel-feed system, using a turbine flow meter, in order to exclude the fuel-feed dynamics from the engine model.

Any alternative gas turbine identification technique must allow the user to assess the accuracy of the resulting models and must not yield lower accuracy than the current procedure. The intention is to improve on the

“wobble” test technique currently used at Rolls Royce through:

- Reduction in the time (and hence cost) of engine testing.
- Improved accuracy of the estimated engine models and an assessment of the model uncertainty.
- Greater insight into the slow engine dynamics.
- Better understanding of the nonlinear behaviour.

The combination of shorter, and thus cheaper, test runs and better coverage of engine behaviour is a strong incentive to look for more efficient test signal designs and model-estimation procedures. The three system identification approaches addressed in this paper are:

- multisine test signals and frequency-domain identification techniques, for improved linear modelling;
- extended least squares with optimal-smoothing, for finding time-varying linear models;
- multiobjective genetic programming, for the selection and identification of a nonlinear model structure.

After a description of each approach and associated results, conclusions will be drawn about the scope and merits of the various techniques.

2. Multisine signals and frequency-domain identification

Previous work by Evans, Rees and Jones (1995) and Evans, Rees and Hill (1998) illustrated how multisine test signals and frequency-domain techniques could be used to accurately estimate parametric and nonparametric models of a gas turbine engine. That work focused on the study of the engine dynamics at a single operating point. In this section, the same techniques are used to estimate *s*-domain models of the engine dynamics at a range of points, with the aim of verifying the linearised thermodynamic models of the engine.

These thermodynamic models are of great importance in the development stages of a gas turbine. Once the turbine is built, it is necessary to check the accuracy of the thermodynamic models with data from the actual engine. The models estimated using frequency-domain techniques have proved to be of great value in this task.

2.1. Multisine testing

It was important to reduce the time spent testing the engine and, to this end, multisine signals were used to excite the gas turbine. These are simply an arbitrary sum of harmonically related cosines

$$u(t) = \sum_{k=1}^F a_k \cos(i_k 2\pi f_0 t + \phi_k) \quad (1)$$

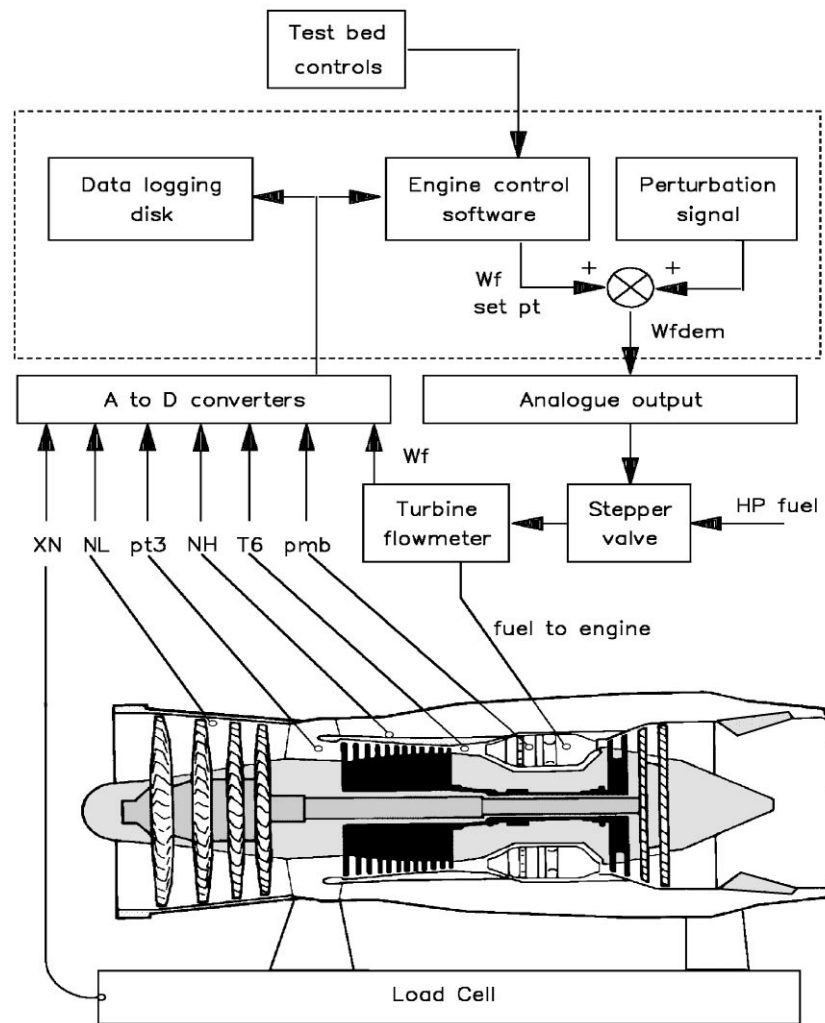


Fig. 2. Engine test facility.

with a_k the cosine amplitude, i_k its harmonic number, f_0 the signal fundamental frequency, ϕ_k the harmonic phase and F the number of frequencies in the signal. The advantages of multisines over other test signals are numerous. The main benefits arise from the fact that multisines can be strictly bandlimited, which is very convenient when no anti-alias filters can be used, and that the vector of harmonics can be arbitrarily defined, which allows the signal power to be placed exactly at the desired frequencies. In addition, a benefit common to all periodic signals is that the nonexcited spectral lines can be eliminated from the data set used for estimation.

By including only odd harmonics in the signal (i.e. the fundamental, the third harmonic, fifth harmonic, and so on) the multisine is made immune to the influence of even-order nonlinearities. All the output frequencies generated by even-order nonlinear terms, such as a squaring function, will fall at even harmonics and not affect the test frequencies (Evans et al., 1995). Such signals are termed

odd-harmonic multisines. The presence of a weak even-order engine nonlinearity had been detected in a previous study (Evans et al., 1995).

The harmonic phases must be carefully selected to minimise the crest factor (CF) of the signal

$$CF = \frac{\max\{|u(t)|\}}{\text{r.m.s.}\{u(t)\}}, \quad (2)$$

which is a measure of the time-domain amplitude compression of the signal for a given *root mean square* (r.m.s.) value. Test amplitudes are often limited by safety or operational considerations (such as preventing surge or engine damage) and also by the need to avoid overly exciting the system nonlinearities. This translates to a set value of $\max\{|u(t)|\}$. With such a limitation, a signal with a low CF will allow more power to be injected into the system and thus improve the signal-to-noise ratio (SNR). The CF minimisation technique proposed by Guillaume,

Schoukens, Pintelon and Kollár (1991) was employed, since it results in the lowest CFs achieved to date.

The use of broad-band multisines allows a large number of frequency points to be measured in one test, which is much faster than measuring frequency by frequency using single sines. This can be illustrated by considering the relative test time with each approach. For single sines, the total test time for a set of F equally spaced measurements, at integer multiples of a fundamental frequency f_0 , will be

$$T_{test} = M_{ss} \frac{1}{f_0} F + T_{set} F = \left(M_{ss} \frac{1}{f_0} + T_{set} \right) F, \quad (3)$$

where M_{ss} is the number of averages and T_{set} is the settling time allowed after each change of frequency before commencing measurements. This expression assumes that each sine is measured for the same time period, in order to ensure equal accuracy at each frequency.

The total test time for a multisine signal will be

$$T_{test} = M_{ms} \frac{1}{f_0} + T_{set}, \quad (4)$$

where M_{ms} is the number of averages of the multisine. Their relative test time can be expressed as

$$\text{Relative } T_{test} = \frac{\text{Multisine } T_{test}}{\text{Single sine } T_{test}} = \frac{M_{ms} T_0 + T_{set}}{(M_{ss} T_0 + T_{set}) F} \quad (5)$$

where T_0 is the period of the fundamental frequency. It is assumed that averaging is performed over the basic interval T_0 in each test. It can be seen that the single sine tests will take F times longer than those with the multisines and that the relative benefit will increase with the number of test frequencies. However, this comparison takes no account of the relative accuracy of the tests. Since the power per harmonic is less for the multisine signal, it may be necessary to perform a greater number of averages in order to achieve an acceptable accuracy ($M_{ms} > M_{ss}$).

Illustrative example: Consider the design of a test to measure the dynamics of the LP shaft, having s -domain poles at $s_1 = -0.45$ and $s_2 = -1.8$, with corresponding time constants $\tau_1 = 2.2$ s and $\tau_2 = 0.55$ s. In order to measure the FRF of this system a frequency range must be selected that adequately covers the system break-points. Since the break-points are at 0.07 and 0.29 Hz then a frequency range of 0.01–0.6 Hz is seen to be adequate. A 30 odd-harmonic multisine with a fundamental frequency of 0.01 Hz has a bandwidth of 0.01–0.59 Hz.

Selecting a settling time of eight times the slowest time constant, the minimum test time for single sines will be

$$\text{Single sine } T_{test} = \left(\frac{1}{0.01} + 8 \times 2.2 \right) \times 30 = 3528 \text{ s.} \quad (6)$$

It was found that measuring six periods of the multisine was sufficient to achieve an acceptable accuracy, giving a test time of

$$\text{Multisine } T_{test} = \left(6 \frac{1}{0.01} \right) + (8 \times 2.2) = 617.6 \text{ s.} \quad (7)$$

This gives a relative test time of

$$\text{Relative } T_{test} = \frac{617.6}{3528} = 0.17, \quad (8)$$

which shows that the use of a multisine signal will reduce the test time by greater than 80%.

A range of multisine signals was used to excite the engine at different operating points. This included the 30 odd-harmonic signal previously mentioned, with power between 0.1 and 0.6 Hz. Signals with power concentrated at low frequencies were also employed, in order to investigate the presence of slow dynamics. The time records of one such signal are shown in Fig. 3, where four periods of the measured fuel flow and the HP and LP shaft speeds are plotted. The spectra of the measured fuel flow and the HP shaft speed are shown in Fig. 4. The input signal consists of odd harmonics between the fundamental and the 71st harmonic, with increased spacing between the higher harmonics. The results are for a test at 75% of the maximum HP shaft speed ($\%N_H$) with an input amplitude of $\pm 10\%$ of the steady-state fuel flow ($\%W_f$). It can be seen that the SNRs are very good for this input amplitude. By excluding the noise lines and averaging over four periods, SNRs of greater than 40 dB were obtained for the input and output.

By omitting all the even harmonics from the input signal, and also excluding some odd harmonics, it is possible to detect the influence of even- and odd-order nonlinearities by the presence of power at the excluded frequencies in the output (Evans, 1998). Any significant nonlinearities in the system will generate contributions at these excluded harmonics, which will rise above the noise floor. No significant components are visible in the output plot, which suggests that the nonlinear influence is small for this input amplitude range.

Tests were conducted at a range of input amplitudes, from $\pm 1\%$ W_f to $\pm 20\%$ W_f . It was found that an amplitude of $\pm 10\%$ represented a good compromise between the need to increase the amplitude in order to improve the SNRs and the need to limit the amplitude in order not to excite the engine nonlinearities. Such a trade-off must be made in testing most real systems.

2.2. Frequency-domain identification

The data obtained from testing with multisine signals were then used to estimate both nonparametric and parametric models in the frequency domain. Since periodic signals were used, the frequency response function (FRF)

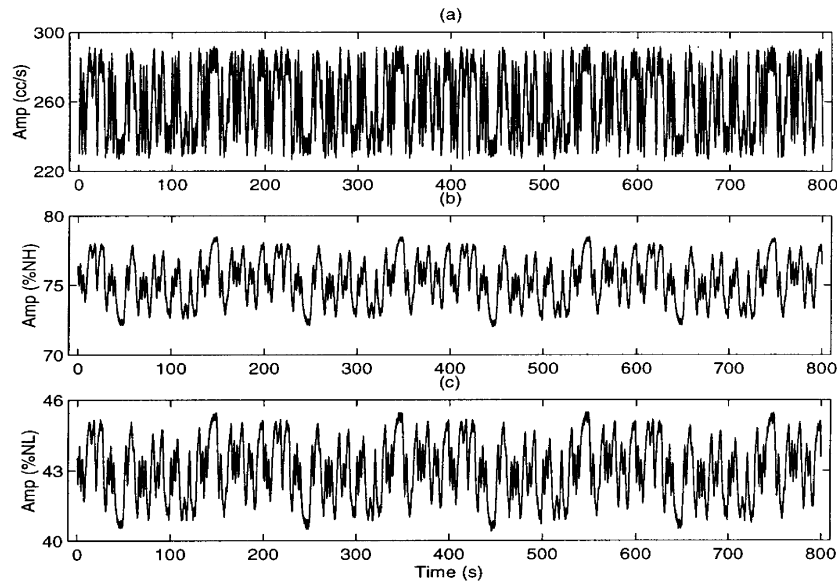


Fig. 3. Time record of the measured (a) fuel flow, (b) HP shaft speed and (c) LP shaft speed.

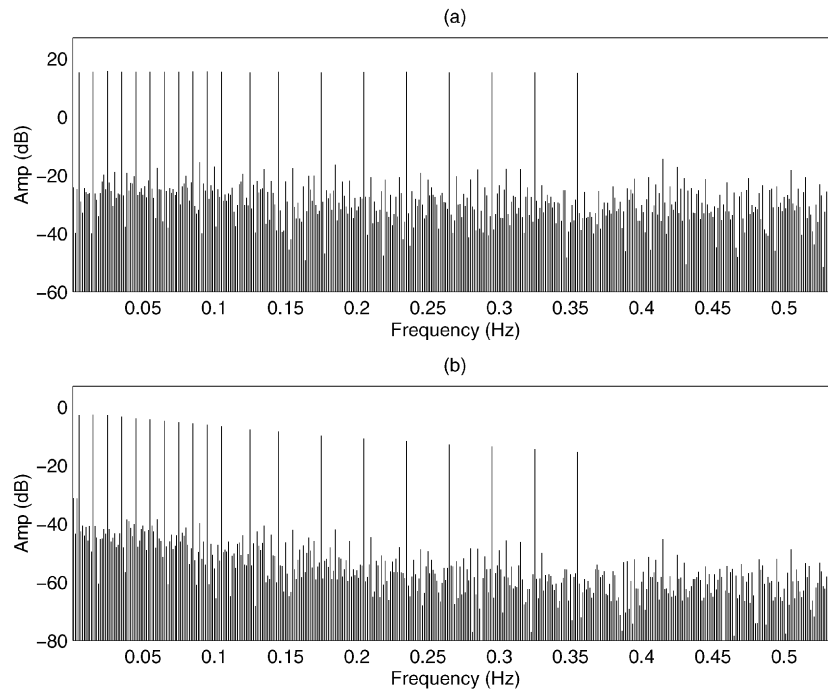


Fig. 4. Spectra of the (a) measured fuel flow and (b) HP shaft speed.

was estimated as the ratio of the mean values of the output and input Fourier coefficients at the discrete test frequencies ω_K

$$\hat{H}_{EV}(j\omega_k) = \frac{(1/M) \sum_{m=1}^M Y_m(j\omega_k)}{(1/M) \sum_{m=1}^M U_m(j\omega_k)}, \quad (9)$$

where $U_m(j\omega_K)$ and $Y_m(j\omega_K)$ are the input and output spectra measured across M periods of the input and

output signals. This was termed the *error-in-variables* estimator by Guillaume (1992), who showed that it is both asymptotically unbiased and efficient in the presence of normally distributed noise on the real and imaginary parts of the input and output Fourier coefficients. It has been shown by Schoukens, Guillaume and Pintelon (1993) that the variance of the estimated FRF is inversely proportional to the number of measurements and the power of the input harmonics, and proportional to the

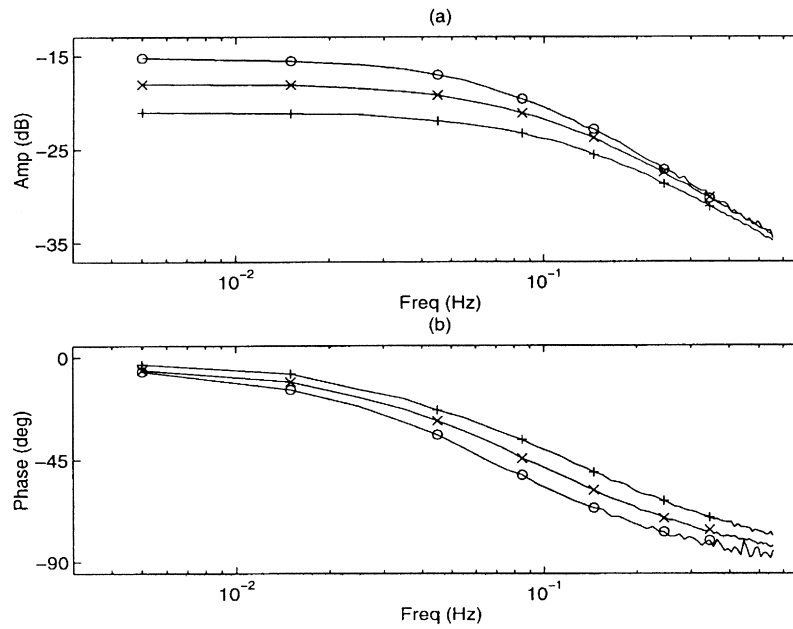


Fig. 5. FRFs for the HP shaft at 65% N_H (o), 75% N_H (x) and 85% N_H (+).

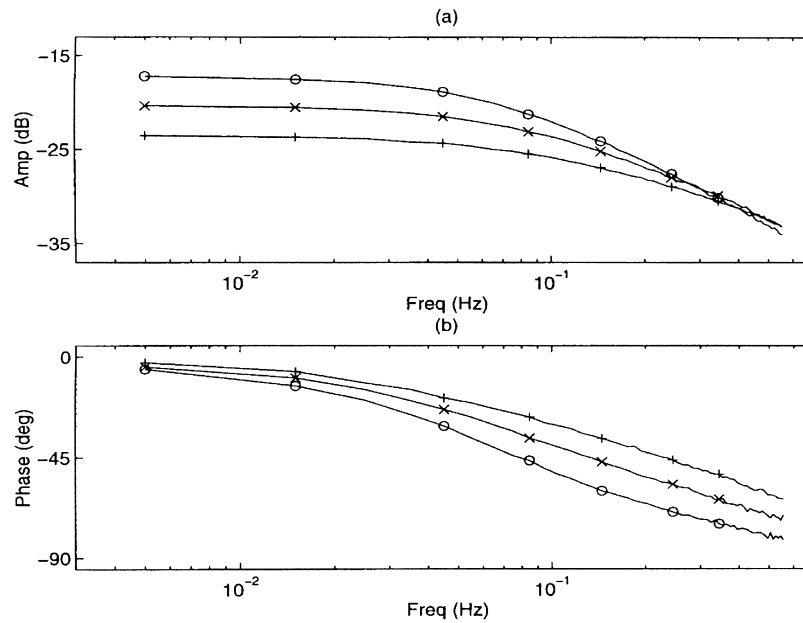


Fig. 6. FRFs for the LP shaft at 65% N_H (o), 75% N_H (x) and 85% N_H (+).

noise variances referred to the system output. With such high SNRs, of greater than 40 dB after averaging, the uncertainty of the FRF estimates was very small.

Fig. 5 shows the HP shaft FRFs resulting from tests at three different operating points, which illustrates the evolution of the shaft dynamics with the shaft speed. Fig. 6 shows the equivalent evolution for the LP shaft. The FRFs give a good insight into the engine dynamics

and they show that both shafts have steady-state gains that decrease, and dynamics that become faster, as the shaft speeds increase.

Parametric frequency-domain identification involves selecting the parameters of an s -domain model:

$$H(s) = \frac{b_0 + b_1 s + \dots + b_m s^m}{a_0 + a_1 s + \dots + a_n s^n} e^{-sT_d}, \quad (10)$$

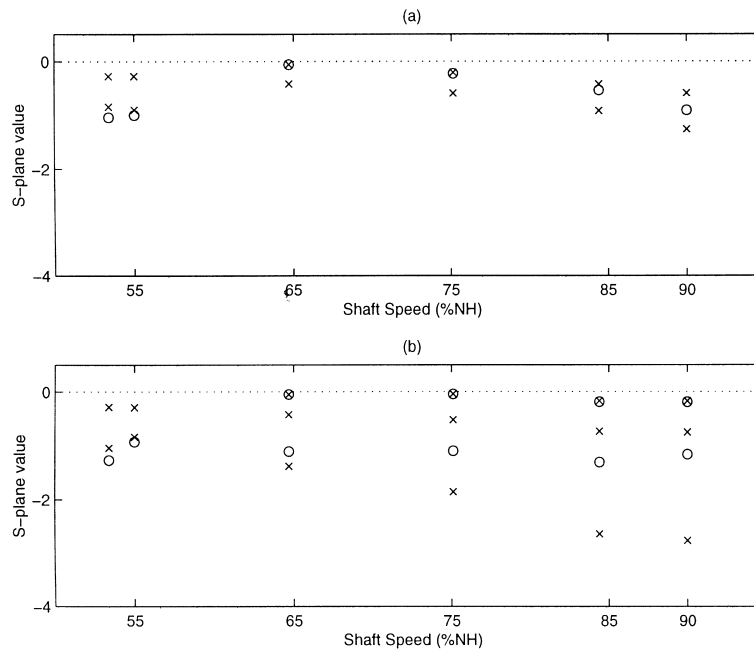


Fig. 7. Evolution of the estimated poles (x) and zeros (o) with operating point: (a) HP shaft and (b) LP shaft.

where m is the number of zeros, n the number of poles and T_d the pure time delay. The estimator developed by Pintelon, Guillaume, Rolain and Verbeyst (1992) and implemented by Kollár (1997) was used for this purpose. The estimator has a least-squares form, with each frequency point weighted by a noise variance term. The pure time delay T_d can also be included as a free parameter for estimation, which is an attractive feature of the frequency-domain approach, since its value is not fixed to multiples of the sampling interval.

A nonparametric noise model is employed and the noise variances and covariance are required as a priori information. These can be estimated from the data before estimation, provided several independent measurements are available. It has been shown that the estimated models are strongly consistent, for an increasing number of data points in each experiment, if the number of independent measurements is greater than or equal to four (Schoukens, Pintelon, Vandersteen and Guillaume, 1997). In the gas turbine testing, each period of the test signal was considered as an independent measurement and at least four periods were measured in each test.

Tests were conducted at several operating points along the turbine running range from 53 to 90% N_H and parametric models of the HP and LP shafts estimated at each point. All of the estimated models had real and negative poles and zeros. The model selection and validation procedure is described in detail in Evans, Rees and Borrell (2000). The locations of the poles and zeros are presented in Fig. 7 and some clear features can be deduced from the plot.

Consider first the tests at 53 and 55% N_H , where the locations of the poles and the zero suggest that the

dynamics are similar for both the HP and LP shafts. At the operating points 65 and 75% N_H the plots suggest a first-order model for the HP shaft dynamics and a second-order model for the LP shaft dynamics. An additional pole-zero pair was found at low frequencies in each model. These low-frequency pole-zero pairs were also found by Evans (1998) when testing the turbine at 75% N_H , where it was suggested that they were modelling a slow dynamic such as heat soakage. Indeed, the time constants associated with this pair are between 5 and 22 s, which are too slow to be modelling the shaft dynamics at these operating points.

At the higher operating points, 85–90% N_H , a different trend is observed for the HP shaft. The low frequency pole and zero have moved apart making the two poles more distinct. In fact, they suggest that the HP shaft behaves like a second-order system at these operating points, as it does at low speeds. This may express the fact that the interaction between the shafts is greater in these regions. The LP shaft is also second order at 85 and 90% N_H but with significantly different dynamics to that of the HP shaft.

2.3. Verification of thermodynamic models

Engine models are required both in the development and operational stages of the life of a gas turbine. Thermodynamic models are derived during the development stage, based on knowledge of the engine physics, and provide important insights into the engine behaviour. Such models are both complex and nonlinear, making them unsuitable for use in the design of engine control systems.

Table 1
Modes of full-order linearised thermodynamic model

S-plane value s	Frequency (Hz)	Time constant (ms)
0.69	0.11	1451
3.1	0.50	319
61	9.7	16
107	17	9
...	...	...
335	53	3
403	64	2

It is therefore common practice to linearise the thermodynamic models around a series of operating points and then carry out a model-order reduction, in order to arrive at models which are suitable for control system design. Since these models are based on a priori assumptions about the engine physics it is important to then validate their performance against real engine data.

In work conducted for Rolls Royce plc, Jackson (1988) showed that, for a given stationary operating point, the higher-order nonlinear thermodynamic models can be reduced to linear models with the same order as the number of engine shafts. The models are first linearised using small perturbations, which in the case of the Spey engine results in models with five inputs, 10 outputs and 15 states. Table 1 shows the four slowest and two fastest modes of such a full-order model, linearised at an operating point of 75% N_H .

There is a big difference between the time constants of the two slowest modes, associated with the shaft dynamics, and the other modes, which are associated with thermodynamic processes in the gas stream. A model reduction procedure can thus be employed to eliminate all of the faster modes. A complete library of such models was generated for the Spey engine, across a range of operating points, by staff at section APD5 (1993) of DERA Pyestock. These linearised, reduced-order models will be referred to simply as the *thermodynamic models* for the remainder of this paper. Evaluating the transfer function matrices of the state space models allows the HP and LP shaft dynamics to be expressed in transfer function form, with common poles but different zeros.

A comparison can now be made between the thermodynamic models and the models estimated directly from engine data. The poles and zeros of the thermodynamic models and the estimated models are plotted together in Fig. 8. In the case of the HP shaft thermodynamic models, the second pole and the zero lie over each other in the range 55–75% N_H . The low-frequency pole-zero pairs of the estimated models have been omitted from this comparison, since they are not related to the shaft dynamics. The plot shows a number of interesting trends. In the first place, there is good agreement between the thermodynamic models and the estimated models with regard to the first-order dynamics at low shaft speeds. This is true for the pole positions of both the HP and LP shafts. A deviation begins to appear at the higher operating points.

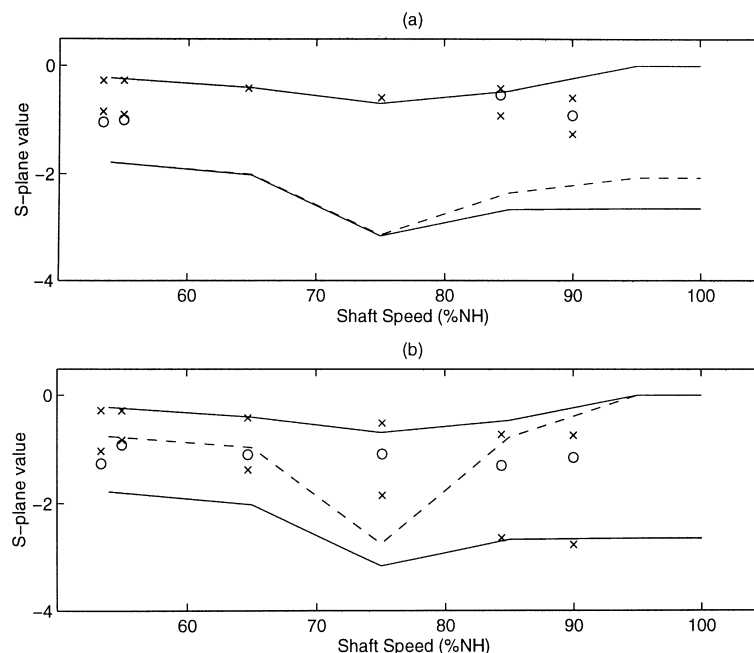


Fig. 8. Poles and zeros of thermodynamic and estimated models for the: (a) HP and (b) LP shafts. Thermodynamic models: poles (solid) and zeros (dashed), and estimated models: poles (x) and zeros (o).

The most notable discrepancy is found in the location of the second pole and the zero of the LP shaft. The thermodynamic models are clearly not modelling the evolution of this pole and zero with shaft speed in the way suggested by the estimated models. The estimated models are based on real data gathered from an engine and the models show close agreement with the engine FRFs estimated at each operating point. It can thus be argued that the estimated models reflect the true dynamics of the engine and that the thermodynamic models, which are based on a priori assumptions about the engine, are problematic. Greater confidence should thus be placed in the experimental results, which clearly show that the thermodynamic models are not adequately representing the dynamics of either shaft at higher operating points. A particular disagreement has been found for the second-order dynamics of the LP shaft, which are of special interest since they model the interaction between the shafts.

It is also interesting to note the behaviour of the estimated HP shaft models at low and high operating points. They suggest that the HP shaft exhibits second-order dynamics in these regions. This second-order behaviour is predicted by the thermodynamic models at the higher operating points but not at the lower.

3. Extended least-squares with optimal smoothing

Traditional single sine engine testing deals with non-linearity only by generating small-signal models about a range of steady-state operating points. It does not cover effects far from equilibrium, which may be prominent in large transients such as slam accelerations (rapid, full throttle opening). Small-signal models for short-term response also give little insight into the slow thermal dynamics of the engine. Although such models are valuable for verifying engine performance near steady state, they are a poor basis for control-system design for large inputs. The main aim here is to investigate large-transient behaviour and slow thermal dynamics and to see whether they can be identified in black-box models from relatively simple large-perturbation tests, economical in test duration.

All records were digitally low-pass filtered with a bandwidth of 1.6 Hz, removing substantial wide-band noise from the flow records, and resampled at an interval of $T = 0.1$ s. The sample means were removed to avoid the need for an offset term. Initial fitting by extended least squares of first- and second-order time-invariant discrete transfer-function models to results of inverse-repeat maximum-length binary sequence (IRMLBS) tests with small ($\pm 10\%$ W_f) perturbation

about high-pressure shaft speeds from 53 to 90% N_H showed:

- poor observability of the small, HP shaft second mode, but a larger, better identified second mode for the LP shaft. Biases are in the order of 10 and 25% in HP and LP dominant time constant if the second mode is omitted, even though that mode is around 10 times faster and has steady-state gain typically 30 times smaller for the HP shaft (although only 3–4 times smaller for the LP shaft at the highest speeds)
- over most of the speed range, lower small-signal dc gain from the transfer-function models than from linearizing the static fuel-flow vs. speed characteristic. Differences are around 10 and 10–20% for HP and LP shafts respectively. The reverse applies at each end of the speed range, by 20–30%. This all suggests that the MLBS tests fail to elicit all the low-frequency response
- smooth variation of the parameters with speed, and low standard errors for the parameters of the dominant mode in second-order models.

For large-transient tests, an optimal-smoothing generalisation of extended least squares (Norton, 1975, 1986) yields time-varying transfer-function models of approximately uniform quality throughout each test. The idea is that the time variation reveals nonlinearity and slow (e.g. thermal) dynamics that would be averaged out by a time-invariant, linear, low-order model. Space only allows reporting of results for N_H .

First, a 254-bit IRMLBS varying N_H by $\pm 10\%$ was superimposed on a triangular wave of the same period, 100 s. This drives N_H between 65 and 85% after several minutes' initial "soaking" at 65% N_H . The most striking feature of the estimates in Figs. 9 and 10 for the pole p and steady-state gain g of the dominant mode is the initial rise in p , rapid during the initial soak (of which 60 s

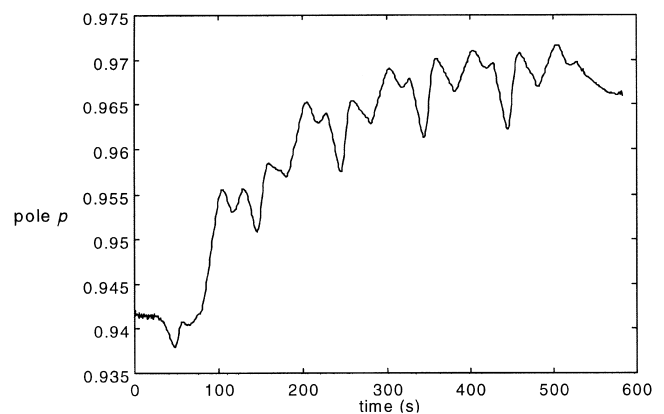
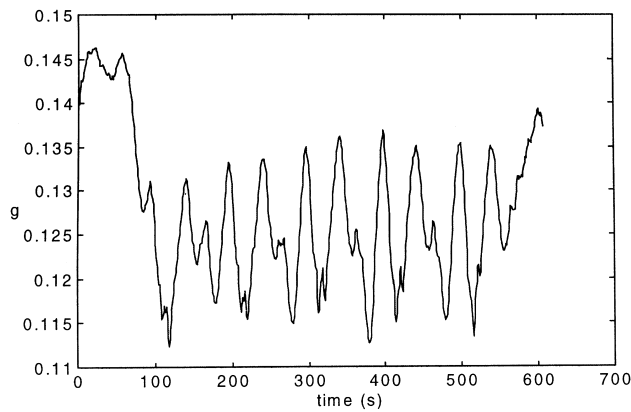
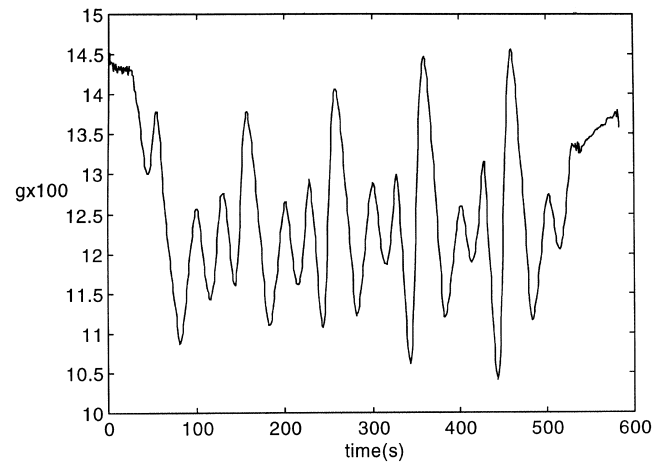
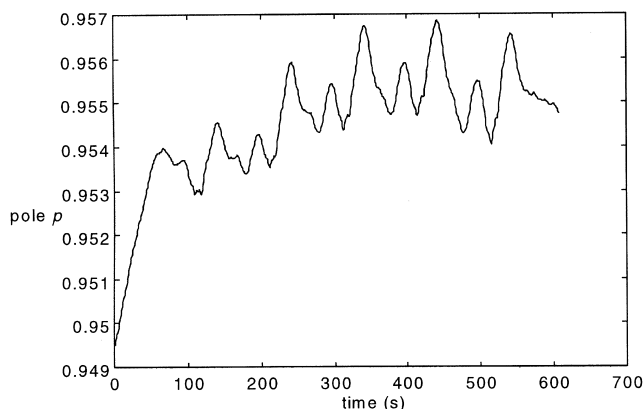


Fig. 9. Dominant discrete pole (p), IRMLBS + triangular wave.

Fig. 10. Dominant mode gain (g), IRMLBS + triangular wave.Fig. 12. Dominant mode gain (g), triangular wave only.Fig. 11. Dominant discrete pole (p), triangular wave only.

is shown) then slower. It is explicable as a slow response to cooling of the engine during the 5-min soak following a test at speeds between 75 and 91%, and implies a long thermal time constant. By contrast, g is near-constant at about the value expected for 65% N_H while soaking, then varies about the value expected at 75% N_H , the mean speed. However, the periodic variation is less than expected from the small-signal results, indicating that thermal equilibrium is never reached.

Another unexpected feature is that the variation of g and, less markedly, p includes higher-frequency components in each perturbation cycle. A possible reason is local structure within the IRMLBS sequence, so comparison with results with no IRMLBS imposed on the triangular wave, Figs. 11 and 12, is called for. Figs. 11 and 12 show smoother variation than Figs. 9 and 10 but similar modulation effects. Clearly dynamics which cannot be represented by moving through a succession of low-order, small-signal models are present. They may be ascribed to higher-order dynamics, perhaps requiring second and/or higher derivatives of speed in the differen-

tial equation describing shaft-speed response to fuel flow, e.g. in the torque equation

$$\frac{\text{power in}}{N_H} = J\dot{N}_H + B(N_H)N_H + \frac{\text{power to exhaust} + \text{heat loss}}{N_H} \quad (11)$$

Such dynamics are ignored in the small-signal models.

A final conclusion to be drawn from Figs. 9 and 11 is that dynamics with a time constant of about 150–200 s are present, evidenced strongly by the overall trend of the pole. These dynamics have a fairly small effect on g but influence the dominant fast-response time constant $-T/\ln p$ by about 30% in Fig. 11.

4. Nonlinear system identification using multiobjective genetic programming

Genetic programming (GP) is an evolutionary paradigm where the computer structures which undergo adaptation are themselves represented as computer programs (Koza, 1992). Koza immediately recognised that symbolic regression was as an obvious application domain, where mathematical expressions and their numeric coefficients may be obtained to provide a good fit to a set of data points. Rodríguez-Vázquez and Fleming (1997) describe how GP can be used to search for a suitable nonlinear model structure for a NARMAX model (Leontaritis and Billings, 1985) of a dynamic system, given an input and output data set. Here, the multiobjective genetic programming (MOGP) approach of Rodríguez-Vázquez and Fleming (1998) is used to obtain NARMAX models to attempt to describe the global behaviour of the gas turbine engine over its operating range. The MOGP fitness function, defined

Table 2
MOGP system identification objectives

Attribute	Objective	Description
Model complexity	1. Model size, NT 2. Degree of nonlinearity, DG 3. Model lag, LG	Number of process and noise terms Maximum order term Maximum lagged input, output and noise terms
Model performance	4. Residual variance, VAR_i	Variance of the predictive error between the predicted and the measured outputs for dataset i
Model validation	5. Long-term prediction error, $LTPE_i$ 6. ACF 7. CCF 8-10 HOC_j as defined in Eq. (16)	Variance of the long-term prediction error for dataset i Auto-correlation, cross-correlation and higher-order correlation-based functions

as a multiobjective function, generates a set of non-dominated models and, thereby, a set of alternative system models. The complexity, performance and also statistical validation of the models are computed for each MOGP population member and contribute to their fitness evaluation. The final set of nondominated model solutions is able to capture the dynamic characteristics of the system and reproduce its behaviour at different operating conditions.

The model representation used is the NARMAX model of Leontaritis and Billings (1985). The general NARMAX model is defined as a nonlinear function of the input, output and noise signal terms as given by the following equation:

$$y(k) = F'(y(k-1), \dots, y(k-n_y), u(k-1), \dots, u(k-n_u), e(k-1), \dots, e(k-n_e)) + e(k), \quad (12)$$

where $y(k)$, $u(k)$ and $e(k)$ represent the output, input and noise signals, respectively, n_y , n_u , and n_e are their associated maximum lags, and F is some unknown nonlinear function of degree ℓ .

Based upon the model representation defined by Eq. (12), the MOGP population consists of tree-structured individuals that readily represent alternative structures for the application of the NARMAX approach. Potential models are encoded as hierarchical tree structures, thus providing a dynamic and variable representation, and these constitute members of a population of different model structures. These structures consist of functions (internal nodes) and terminals (leaf nodes) appropriate to the problem domain. Hence, the function set is here defined as $F = \{\text{ADD}, \text{MULT}\} = \{+, *\}$, and the terminal set as $T = \{X_0, \dots, X_{n_y}, X_{n_y+1}, \dots, X_{n_y+n_u}, X_{n_y+n_u+1}, \dots, X_{n_y+n_u+n_e}\} = \{c, y(k-1), \dots, y(k-n_y), u(k-1), \dots, u(k-n_u), e(k-1), \dots, e(k-n_e)\}$. For example, one hierarchical tree representation

of the polynomial NARMAX model might be expressed in Polish notation as $(\text{ADD}(\text{ADD}(\text{ADD } X1 \ X4) \ X5)(\text{MULT}(\text{ADD } X2 \ X3)(\text{ADD } X1 \ X2)))$. This is equivalent to the polynomial nonlinear model defined as

$$y(k) = \theta_0 + \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 e(k-1) + \theta_5 y(k-1)^2 + \theta_6 y(k-1)y(k-2) \quad (13)$$

where $\{X1, X2, X3, X4, X5\} = \{1.0(\text{the constant term}), y(k-1), y(k-2), u(k-1), e(k-1)\}$. A least-squares algorithm is applied to compute the parameters, θ_i , to minimise the residual of errors, ε , between the measured output $y(k)$ and the predicted output $\hat{y}(k)$ that is given by

$$\varepsilon(k) = y(k) - \hat{y}(k, \hat{\theta}). \quad (14)$$

In order to perform selection at each generation in the MOGP approach, all model measures considered in the identification are evaluated for each member of the population. The fitness value of each population member is assigned by means of a rank-based fitness method (Fonseca and Fleming, 1998). This fitness evaluation is based on the definition of Pareto-optimality or nondominance. If we consider a minimisation problem and, given two n -component objective function vectors, $\mathbf{f}_u$ and $\mathbf{f}_v$, we can say that $\mathbf{f}_u$ dominates $\mathbf{f}_v$ (is Pareto-optimal) if

$$\forall i \in \{1, \dots, n\}, f_{u_i} \leq f_{v_i} \wedge \exists i \in \{1, \dots, n\}, f_{u_i} < f_{v_i}, \quad (15)$$

producing a set of possible and valid solutions known as the Pareto-optimal or nondominated set. That is, a Pareto-optimal solution is one in which an improvement in one of the design objectives will lead to degradation in one or more of the remaining objectives.

Thus, the model complexity, performance and validation attributes of the modelling process can

simultaneously be evaluated by considering them in the multiobjective fitness function. The objectives used in this study are defined and classified as shown in Table 2. The validation attributes are measured, based upon the auto-correlation (ACF), cross-correlation (CCF) and higher-order correlation (HOC) functions proposed by (Billings & Voon, 1986) and defined as

$$\text{ACF} = \Phi_{ee}(\tau) = E[\varepsilon(k - \tau)\varepsilon(k)] = \delta(\tau),$$

$$\text{CCF} = \Phi_{ue}(\tau) = E[u(k - \tau)\varepsilon(k)] = 0 \quad \forall \tau,$$

$$\text{HOC}_1 = \Phi_{\varepsilon(u)}(\tau) = E[\varepsilon(k)\varepsilon(k - \tau)u(k - \tau)] = 0, \quad \tau \geq 0,$$

$$\text{HOC}_2 = \Phi_{u^2\varepsilon}(\tau) = E[\{u^2(k) - E[u^2(k)]\}\varepsilon(k - \tau)] = 0 \quad \forall \tau,$$

$$\text{HOC}_3 = \Phi_{u^2\varepsilon^2}(\tau) = E[\{u^2(k) - E[u^2(k)]\}\varepsilon^2(k - \tau)] = 0 \quad \forall \tau.$$

(16)

The objectives related to the performance and complexity of candidate NARMAX models are shown in Table 2.

The identification approach presented here concerns the identification of nonlinear models that can cope with a specific range of system operation. As such, it is based upon data obtained from multisine excitation signals at different operating conditions. The objective is to find a set of NARMAX models that minimises the n -components vector criterion which includes measures of variance and LTPE at these different operating conditions, albeit, operating within a specific working region. The multiobjective vector function to be minimised is defined as:

$$\mathbf{f} = [\text{NT}, \text{DG}, \text{LG}, \text{VAR}_i, \text{LTPE}_i, \text{ACF}, \text{CCF}, \text{HOC}_j]^T$$

where the variables are defined in Table 2.

Initial runs that were conducted with a multiobjective vector function

$$\mathbf{f} = [\text{NT}, \text{DG}, \text{LG}, \text{VAR}_i, \text{LTPE}_i]^T$$

which did not include model validation criteria, produced a set of linear models. However, these solutions failed to satisfy validation requirements. Consideration of the full objective function vector produced models containing quadratic terms. Since multiobjective optimisation produces a family of solutions, the MOGP procedure invites designer interaction. This resulted in three candidate solutions, whose term content is given in Table 3. Note that even though the terminal set T included past values of the residuals $\{e(k - 1), \dots, e(k - n_e)\}$, no NARMAX structures emerged; instead, only NARX model structures were produced. These three models were subjected to rigorous simulation tests (Rodríguez-Vázquez 1999) and Model 3 was selected as the most appropriate. The structure and parameters of

Table 3
Quadratic polynomial models

Model term	Model		
	1	2	3
c	▲	▲	▲
$y(k - 1)$	▲	▲	▲
$y(k - 2)$	▲	▲	▲
$u(k - 1)$			▲
$u(k - 2)$			▲
$y(k - 1)^2$			▲
$y(k - 2)^2$	▲		
$y(k - 1)u(k - 1)$	▲	▲	
$y(k - 2)u(k - 2)$	▲	▲	

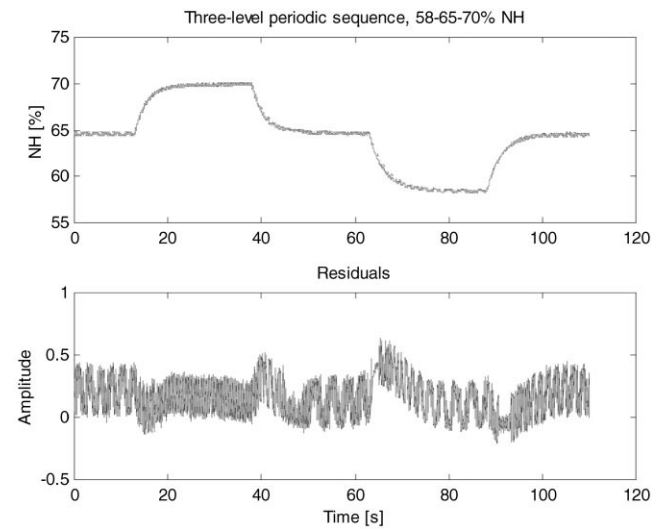


Fig. 13. (a) Three-level periodic sequence of period 100 s, each period consisting of quarter-periods at levels 65, 70, 65 and 58% N_H (actual: solid line, estimated: dotted line). (b) Corresponding residuals.

this quadratic NARX engine model are

$$\begin{aligned} y(k) = & -1.16236 \times 10^0 + 7.96090 \times 10^{-1}y(k - 1) \\ & + 2.39363 \times 10^{-1}y(k - 2) \\ & + 4.33425 \times 10^{-3}u(k - 1) \\ & - 5.90401 \times 10^{-4}u(k - 2) \\ & \times 4.37337 \times 10^{-4}y(k - 1)^2. \end{aligned} \quad (17)$$

The ability of this model to represent the engine relationship between the measured fuel flow and the high pressure shaft speed on two exacting test scenarios is illustrated in Fig. 13 (three-level periodic signals over the range 58–65–70% N_H) and Fig. 14 (a large transient

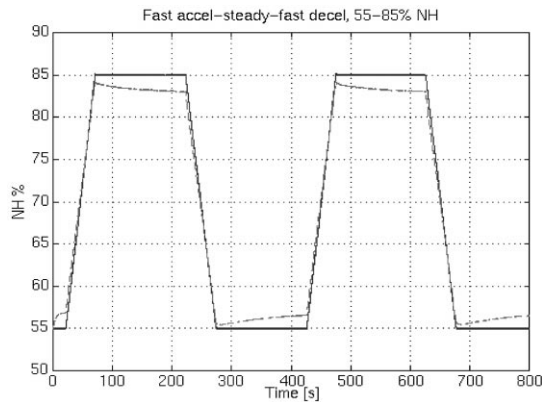


Fig. 14. Fast acceleration/deceleration signal (55–85% N_H), actual: solid line, estimated: dotted line.

55–85% N_H). The upper plot in Fig. 13 shows the two plots of actual and estimated data (nearly superimposed) and the lower plot displays the residuals (difference between the actual and estimated data) which are very small. Recall that the objectives directed the search to find good low-order simple non-linear models. Fuller results are provided in Rodríguez-Vázquez (1999).

5. Conclusions

The three techniques outlined in this paper were all applied to real engine test data. They provide significant insights into alternative identification strategies. The aims, to improve the efficiency of gas turbine dynamic testing and to see how far the identification approaches can give insight into complex engine behaviour, have largely been realised. The conclusions from each of the techniques will be summarised in turn.

The contribution based on the use of multisine signals and frequency-domain techniques has been:

- The design of multisine signals with a low crest factor to excite the dynamics of the gas turbine. These multisines allow a very significant reduction in test time, compared with the single-sine “wobble” tests currently used, without any significant loss of accuracy. Reductions in test times of 80%, or greater, can be achieved by using multisine signals.
- The levels of noise and nonlinearities present in the turbine have been assessed for small-signal tests. For an input signal amplitude of $\pm 10\% W_f$, it was found that the SNRs were better than 40 dB, after averaging, and that the nonlinear effects were negligible.
- The nonparametric frequency response functions have been estimated at various operating points and their confidence bounds have been determined. These were found to be very small for an input signal amplitude of

$\pm 10\% W_f$, providing a high degree of confidence in the estimated frequency responses. Parametric s -domain models have been estimated directly in the frequency domain and the uncertainty on their poles and zeros was found to be small. The pure time delay was included as an estimated parameter and the results matched those predicted by theory (see Evans et al., 2000).

- The parametric models estimated from the engine data were then used to verify the linearised thermodynamic models derived from the engine physics. This showed significant discrepancies between the estimated and derived models, which warrant further investigation. This was particularly true for the second-order dynamics of the LP shaft and the HP shaft at the lower and higher operating points.

The extended least squares with optimal-smoothing approach allowed investigation of engine dynamics over large perturbations. Hitherto, gas turbine identification techniques have concentrated on capturing dynamics using small-signal models. Evidence of slow thermal dynamics, with implications for engine test duration, was found using this approach.

In contrast to the second technique, which was concerned with time-varying linear models, the last technique employed multiobjective genetic programming for the identification of time-invariant nonlinear (NARMAX) models. The multiobjective approach permitted the simultaneous evaluation of multiple independent criteria in the identification procedure and genetic programming was used to conduct the search over a range of nonlinear terms. The use of model complexity objectives alongside model performance objectives supported the identification of low-order nonlinear models for a specific large-amplitude operating region. The validation functions (correlation tests) led to the identification of second-order dynamic models containing a quadratic nonlinear term. By enabling designer interaction in the process, it was possible to selectively reduce the search space while pursuing a set of objectives. This focused the genetic search into a region of acceptable models. Validation of the identified models against differing data sets proved the acceptability of the identified models.

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